6105. a) $z = e^{i\pi/4} = \cos\frac{\pi}{4} + i\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} + \frac{i\sqrt{2}}{2} = \frac{\sqrt{2}(1+i)}{2}$

b) $z = e^{i\pi} = \cos\pi + i\sin\pi = -1 + 0 = -1$

c) $z = e^{-i\pi/3} = \cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right) = \cos\frac{\pi}{3} - i\sin\frac{\pi}{3} =$

$= \frac{1}{2} - \frac{i\sqrt{3}}{2} = \frac{1-i\sqrt{3}}{2}$

d) $z = e^{i7\pi/6} = \cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6} = -\frac{\sqrt{3}}{2} - \frac{i}{2} = -\frac{\sqrt{3}+i}{2}$

Svar: a) $\frac{\sqrt{2}(1+i)}{2}$ b) -1 c) $\frac{1-i\sqrt{3}}{2}$ d) $-\frac{\sqrt{3}+i}{2}$

6106. $z = e^{i\pi/6} = \cos\frac{\pi}{6} + i\sin\frac{\pi}{6} = \frac{\sqrt{3}}{2} + \frac{i}{2} = \frac{\sqrt{3}+i}{2}$

$\bar{z} = \frac{\sqrt{3}-i}{2} = e^{-i\pi/6}$

a) $z + \bar{z} = \frac{\sqrt{3}+i}{2} + \frac{\sqrt{3}-i}{2} = \sqrt{3}$

b) $z^3 = (e^{i\pi/6})^3 = e^{i3\pi/6} = e^{i\pi/2} =$

$= \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} = 0 + i = i$

c) $z \cdot \bar{z} = |z|^2 = 1^2 = 1$

d) $\frac{z}{\bar{z}} = \frac{e^{i\pi/6}}{e^{-i\pi/6}} = e^{2i\pi/6} = e^{i\pi/3} = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3} =$

$= \frac{1}{2} + \frac{i\sqrt{3}}{2} = \frac{1+i\sqrt{3}}{2}$

Svar: a) $\sqrt{3}$ b) i c) 1 d) $\frac{1+i\sqrt{3}}{2}$

6107. $z_1 = e^{i\pi/2} = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} = 0 + i = i$

$z_2 = e^{i\pi/3} = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3} = \frac{1}{2} + \frac{i\sqrt{3}}{2} = \frac{1+i\sqrt{3}}{2}$

a) $z_1 + z_2 = i + \frac{1+i\sqrt{3}}{2} = \frac{2i}{2} + \frac{1+i\sqrt{3}}{2} = \frac{1+(\sqrt{3}+2)i}{2}$

b) $z_1 \cdot z_2 = i \cdot \frac{1+i\sqrt{3}}{2} = \frac{i+i^2\sqrt{3}}{2} = \frac{-\sqrt{3}+i}{2}$

c) $\frac{z_1}{z_2} = \frac{e^{i\pi/2}}{e^{i\pi/3}} = e^{i\pi/2 - i\pi/3} = e^{3i\pi/6 - 2i\pi/6} = e^{i\pi/6} =$

$= \cos\frac{\pi}{6} + i\sin\frac{\pi}{6} = \frac{\sqrt{3}}{2} + \frac{i}{2} = \frac{\sqrt{3}+i}{2}$

d) $(z_1 + z_2)^2 = \left(\frac{1+i(\sqrt{3}+2)}{2}\right)^2 = \frac{(1+i(\sqrt{3}+2))^2}{4} =$

$= \frac{1^2 + 2i(\sqrt{3}+2) + i^2(\sqrt{3}+2)^2}{4} =$

$= \frac{1 + 2i\sqrt{3} + 4i - (\sqrt{3}+2)^2}{4} = \frac{1 + 2i\sqrt{3} + 4i - 3 - 4\sqrt{3} - 4}{4} =$

$= \frac{-6 - 4\sqrt{3} + 2i(\sqrt{3}+2)}{4} = \frac{-(3+2\sqrt{3}) + (\sqrt{3}+2)i}{2}$

Svar: a) $\frac{1+(\sqrt{3}+2)i}{2}$ b) $\frac{-\sqrt{3}+i}{2}$ c) $\frac{\sqrt{3}+i}{2}$

d) $\frac{-(3+2\sqrt{3}) + (\sqrt{3}+2)i}{2}$

6108. a) $e^{i\pi/2} + e^{-i\pi/2} =$

$= \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} + \cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right) =$

$= \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} + \cos\frac{\pi}{2} - i\sin\frac{\pi}{2} = 2\cos\frac{\pi}{2} = 2 \cdot 0 = 0$

b) $2e^{i5\pi/4} - 2e^{-i5\pi/4} =$

$= 2\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right) - 2\left(\cos\left(-\frac{5\pi}{4}\right) + i\sin\left(-\frac{5\pi}{4}\right)\right) =$

$= 2\cos\frac{5\pi}{4} + 2i\sin\frac{5\pi}{4} - 2\cos\frac{5\pi}{4} + 2i\sin\frac{5\pi}{4} = 4i\sin\frac{5\pi}{4} =$

$= 4i \cdot \left(-\frac{\sqrt{2}}{2}\right) = -2i\sqrt{2}$

Svar: a) 0 b) $-2i\sqrt{2}$

6109. $(e^{i\pi/2} + e^{i4\pi/3}) = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} + \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3} =$

$= 0 + i - \frac{1}{2} - \frac{i\sqrt{3}}{2} = \frac{2i}{2} - \frac{1}{2} - \frac{i\sqrt{3}}{2} = \frac{-1+(2-\sqrt{3})i}{2}$

Svar: $\frac{-1+(2-\sqrt{3})i}{2}$

6110. $e^{a+ib} = e^a \cdot e^{ib}$

$e^{ib} = \cos b + i\sin b$

$|e^{ib}| = |\cos b + i\sin b| = \sqrt{\cos^2 b + \sin^2 b} = \sqrt{1} = 1$

(Trigonometriska ettan)

$|e^{a+ib}| = |e^a \cdot e^{ib}| = |e^a| \cdot |e^{ib}| = e^a \cdot 1 = e^a$

Svar: e^a

6111. $e^{(1+i)v} + e^{(1-i)v} = e^{v+iv} + e^{v-iv} = e^v \cdot e^{iv} + e^v \cdot e^{-iv} =$

$= e^v \cdot (\cos v + i\sin v) + e^v \cdot (\cos(-v) + i\sin(-v)) =$

$= e^v \cos v + ie^v \sin v + e^v \cos v - ie^v \sin v = 2e^v \cos v$

Svar: $2e^v \cos v$

6112. $e^{i\pi/2} = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} = 0 + i = i$

Vi kan alltså skriva $i^i = \left(e^{i\pi/2}\right)^i = e^{i^2\pi/2} = e^{-\pi/2}$
där vi använt potenslagarna.

Svar: $e^{-\pi/2}$

6113. a) $e^{|i|} = e^1 = e$

b) $e^{|i^2|} = e^{|-1|} = e^1 = e$

c) $|2+i| = \sqrt{2^2 + 1^2} = \sqrt{5}$

$e^{|2+i|} = e^{\sqrt{5}}$

d) $|\cos v + i\sin v| = \sqrt{\cos^2 v + \sin^2 v} = \sqrt{1} = 1$

(Trigonometriska ettan)

$e^{|\cos v + i\sin v|} = e^1 = e$

Svar: a) e b) e c) $e^{\sqrt{5}}$ d) e

6114. Se lösning i lärobokens facit.